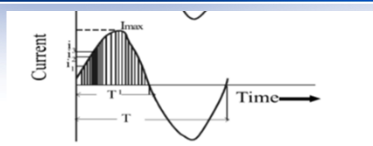
AC Circuits
Basics of Electrical Engineering

RMS Value (Root Mean Square Value)



heat energy produced during 1st interval = $i_1^2 R \frac{T'}{n}$ joule

heat energy produced during 2nd interval = $i_2^2 R \frac{T'}{n}$ joule

heat energy produced during the nth interval = $i_n^2 R \frac{T'}{n}$ joule

Total heat energy produced in the all n intervals $W = RT' \frac{(i_1^2 + i_2^2 + \dots + i_n^2)}{n}$

$W = I^2 RT'$ joule

RMS Value (Root Mean Square Value)

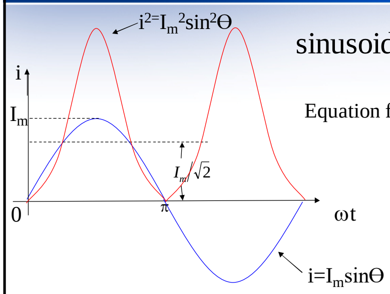
$$I^2 RT' = RT' \frac{i_1^2 + i_2^2 + \cdots + i_n^2}{n}$$

$$I^2 = \frac{i_1^2 + i_2^2 + \cdots + i_n^2}{n}$$ = mean of square of instantaneous currents

$$I = \sqrt{\frac{i_1^2 + i_2^2 + \cdots + i_n^2}{n}}$$ = square root of mean of squares of instantaneous currents = Root mean square (RMS) value of current

$$E = \sqrt{\frac{(e_1^2 + e_2^2 + \cdots + e_n^2)}{n}}$$

RMS Value – Integral method



sinusoidal current $i = I_m \sin\theta$

Equation for squared wave $i^2 = I_m^2 \sin^2\theta$

RMS Value

The mean of the squares of instantaneous value of currents over the first half cycle = area of the first half cycle of squared wave ÷ Its base

$$= \frac{\int_0^{\pi} i^2\, d\theta}{\pi} = \frac{1}{\pi}\int_0^{\pi} (I_m^2\, Sin^2\theta)\, d\theta$$

$$= \frac{I_m^2}{\pi}\int_0^{\pi}\left(\frac{1-Cos\,2\theta}{2}\right) d\theta = \frac{I_m^2\left[\theta - \frac{Sin\,2\theta}{2}\right]_0^{\pi}}{2\pi}$$

$$= \frac{I_m^2}{2\pi} \times \pi = \frac{I_m^2}{2}$$

root of mean of squares (rms value) $I = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$

RMS Value

$$I_{RMS} = \sqrt{\frac{1}{T}\int_0^T i^2 d\theta}$$

$$E_{RMS} = \sqrt{\frac{1}{T}\int_0^T e^2 d\theta}$$

Average Value

- Average value of an alternating quantity is defined as that steady direct current which transfers across any circuit the same amount of charge as is transferred by the alternating current during the same time

- 1) Mid ordinate method

$$I_{avg} = \frac{i_1 + i_2 + i_3 + \ldots + i_n}{n}$$

Average Value- Analytical Method

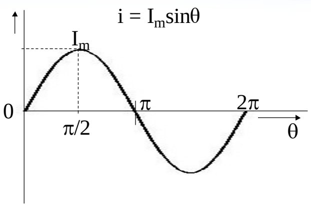
sinusoidal current $i = I_m \sin\theta$

$$I_{av} = \frac{\text{Area under half cycle}}{\pi}$$

$$= \frac{\int_0^{\pi} i\, d\theta}{\pi}$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m\, Sin\,\theta\, d\theta$$

$$= \frac{I_m}{\pi}[-\,Cos\,\theta]_0^{\pi} = \frac{2I_m}{\pi}$$



$$I_{av} = 0.637\, I_m$$

Peak Factor/ Amplitude Factor (K_p)

$$Peak\ factor = \frac{Maximum\ value}{RMS\ value}$$

$$= \sqrt{2}\ \text{for sinusoidal}$$

$$= 1.414$$

Form Factor (K_f)

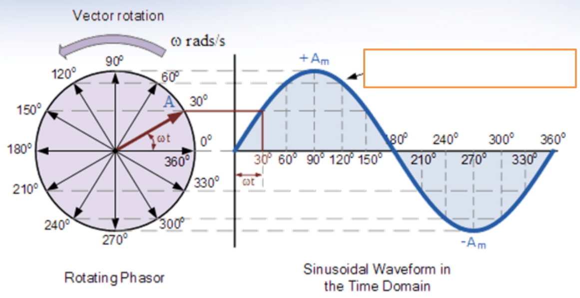
$$Form\ Factor = \frac{RMS\ value}{Average\ value}$$

$$= 1.11\ \text{for sinusoidal}$$

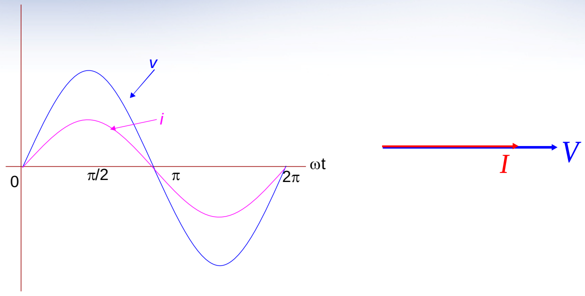
Tutorial

Given $i = 62.35\sin 323t$ A. Find f, I, I_m, I_{av}, K_f

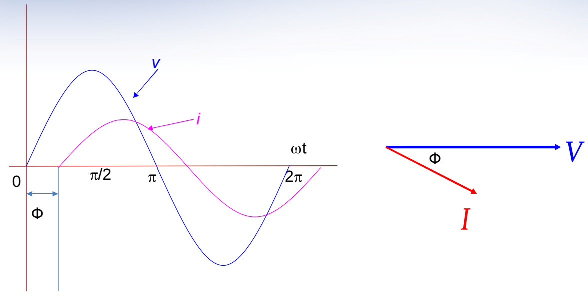
Phasor



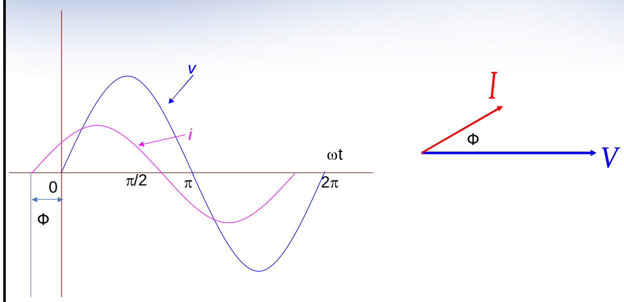
In phase quantities



Lagging current



Leading current



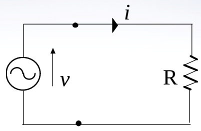
- ❖ Phasors which reaches the vertical position first in the assumed direction of rotation is called leading

AC Through a Purely Resistive Circuit

Let

$v = V_m \sin \omega t$ --------------- **(1)**

$i = \frac{v}{R}$



AC Through a Purely Resistive Circuit

$$i = \frac{V_m}{R} \sin \omega t$$

i is maximum when sinωt is unity

ie

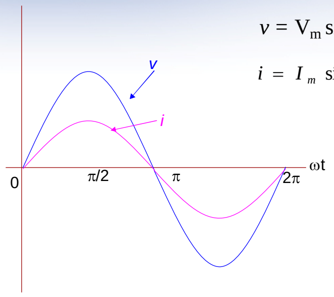
$$i = I_m \sin \omega t$$

$$I_m = \frac{V_m}{R}$$

$$I_{rms} = \frac{V_{rms}}{R}$$

$$V = IR$$

AC Through a Purely Resistive Circuit



$v = V_m \sin \omega t$ --------------- **(1)**

$i = I_m \sin \omega t - - - - - - (2)$

Power in Purely R Circuit

$$p = vi$$

$$p = vi = V_m \sin \omega t \times I_m \sin \omega t$$

$$p = V_m I_m \sin^2 \omega t$$

$$p = \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$

$$p = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$$

Power consists of

- Constant part $\frac{V_m I_m}{2}$
- fluctuating part $\frac{V_m I_m}{2} \cos 2\omega t$

Power in Purely R Circuit

- For complete cycle, average value $\frac{V_m I_m}{2}\cos 2\omega t$ is zero

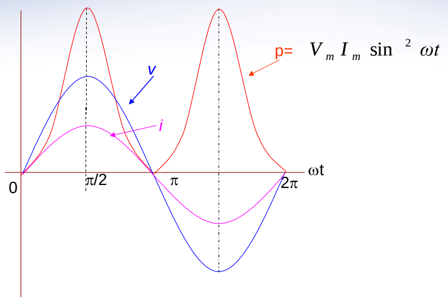
- Power for whole cycle $= \frac{V_m I_m}{2}$

$$= \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}$$

$$= V_{rms} I_{rms}$$

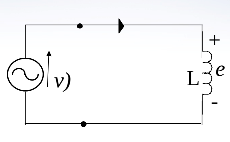
$$= VI = \frac{V^2}{R} = I^2 R$$

Power in Purely R Circuit



AC Through Purely Inductive Circuit

$v = V_m \sin \omega t$



Due to the inductance of the coil, a self induced emf $-L\frac{di}{dt}$ is induced in the coil which opposes the applied voltage at every instant

AC Through Purely Inductive Circuit

$v = V_m \sin \omega t$ 1

$v = L\frac{di}{dt}$

$V_m \sin \omega t = L\frac{di}{dt}$

$di = \frac{V_m}{L}\sin \omega t \; dt$

$i = \frac{V_m}{L}\int \sin \omega t \; dt$

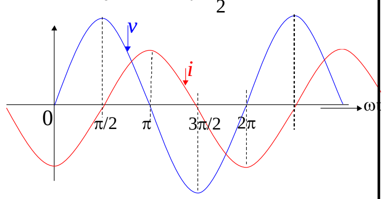
$= \frac{V_m}{\omega L}(-\cos \omega t)$

$i = \frac{V_m}{\omega L}\sin(\omega t - \frac{\pi}{2})$

$i = I_m \sin(\omega t - \frac{\pi}{2})$........2

where $I_m = \frac{V_m}{\omega L}$

From equ (1) and (2) it is clear that I lags behind v by $\frac{\pi}{2}$



AC Through Purely Inductive Circuit

- Current through pure inductor lags the voltage across it by 90°.

$V_m = \omega L I_m$

$= X_L I_m$

$X_L = \omega L = 2\pi f L$ ----- **inductive reactance, in ohm**

$I_m = \frac{V_m}{X_L}$ $\quad \frac{I_m}{\sqrt{2}} = \frac{V_m/\sqrt{2}}{X_L}$ $\quad \mathbf{I = \frac{V}{X_L}}$ $\quad \mathbf{X_L = \frac{V}{I}}$

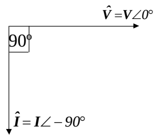
AC Through Purely Inductive Circuit

$v = V_m \sin \omega t$ 1

$i = I_m \sin(\omega t - \frac{\pi}{2})$........2

Phasor Diagram

$\frac{\bar{V}}{\bar{I}} = \frac{V\angle 0}{I\angle -90^0} = jX_L$ where $\frac{V}{I} = X_L$



Power in Purely L Circuit

$p = vi$

$p = vi = V_m \sin \omega t \times I_m \sin(\omega t - \frac{\pi}{2})$

$p = -V_m I_m \sin \omega t \cos \omega t$

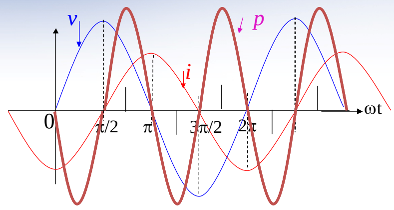
Power for complete cycle

$P = -\frac{V_m I_m}{2} \int_0^{2\pi} \sin 2\omega t$

$p = -\frac{V_m I_m}{2} \left[\frac{-\cos 2\omega t}{2} \right]_0^{2\pi} = 0$

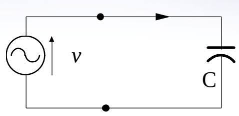
Average power demand is zero. However Maximum value of instantaneous power is $\frac{V_m I_m}{2}$

Power in Purely L Circuit



AC Through Purely Capacitive Circuit

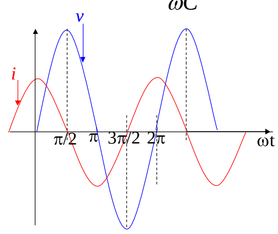
- v- pd developed between the plates of capacitor at any instant
- q- Charge on plates at that instant
- q= Cv, where C is the capacitance



AC Through Purely Capacitive Circuit

$$q = Cv$$
$$v = V_m \sin \omega t ....(1)$$
$$q = CV_m \sin \omega t$$
$$i = \frac{dq}{dt}$$
$$i = \frac{d}{dt}(CV_m \sin \omega t)$$
$$i = \omega C V_m (\cos \omega t)$$
$$i = \frac{V_m}{1/\omega C} \sin(\omega t + \frac{\pi}{2})$$

$$i = I_m \sin(\omega t + \frac{\pi}{2}) ......(2)$$
$$I_m = \frac{V_m}{1/\omega C} = \frac{V_m}{X_c}$$
$$X_c = \frac{1}{\omega C}$$



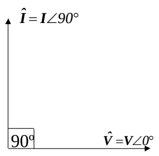
AC Through Purely Capacitive Circuit

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$I_m = \frac{V_m}{X_C} \qquad \frac{I_m}{\sqrt{2}} = \frac{V_m/\sqrt{2}}{X_C} \qquad I = \frac{V}{X_C} \qquad X_C = \frac{V}{I}$$

- Comparing equations (1) and (2) it is clear that i is leading v by 90^0

$$\frac{\bar{V}}{\bar{I}} = \frac{V\angle 0}{I\angle 90^0} = -jX_C \quad \text{where} \quad \frac{V}{I} = X_C$$



Power in Purely C Circuit

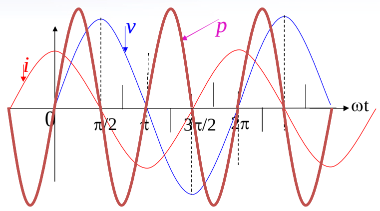
$$p = vi$$
$$p = vi = V_m \sin \omega t \times I_m \sin(\omega t + \frac{\pi}{2})$$
$$p = V_m I_m \sin \omega t \cos \omega t$$

Power for complete cycle

$$P = \frac{V_m I_m}{2} \int_0^{2\pi} \sin 2\omega t$$
$$p = \frac{V_m I_m}{2} \left[\frac{-\cos 2\omega t}{2} \right]_0^{2\pi} = 0$$

Average power demand is zero. However Maximum value of instantaneous power is $\frac{V_m I_m}{2}$

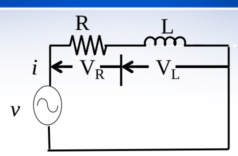
Power in Purely C Circuit

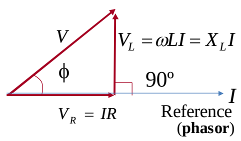


CIVIL

AC through Series RL Circuit

- V
- I
- $V_R = IR$
- $V_L = IX_L$





AC through Series RL Circuit

$$V = \sqrt{V_R^2 + V_L^2}$$

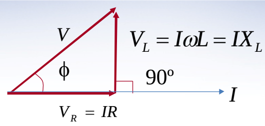
$$V = \sqrt{(IR)^2 + (IX_L)^2}$$

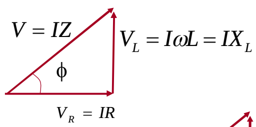
$$V = I\sqrt{(R)^2 + (X_L)^2}$$

$$V = IZ$$

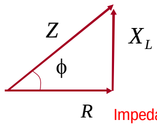
$$Z = \sqrt{(R)^2 + (X_L)^2}$$

$$\tan\phi = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{\text{Reactance}}{\text{Resistance}}$$





Voltage Triangle

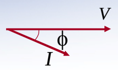


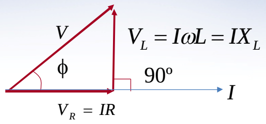
Impedance Triangle

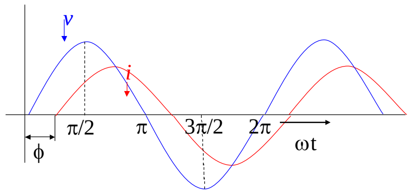
AC through Series RL Circuit

$v = V_m \sin \omega t$

$i = I_m \sin(\omega t - \phi)$







AC through Series RL Circuit

- Instantaneous power consumed $p = vi$

$$p = V_m \sin \omega t I_m \sin(\omega t - \phi)$$

$$p = V_m I_m \sin \omega t \sin(\omega t - \phi)$$

$$p = \frac{V_m I_m}{2}\left[\cos(\omega t - \omega t + \phi) - \cos(2\omega t - \phi)\right]$$

$$p = \frac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t - \phi)\right]$$

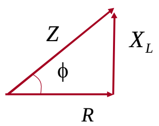
Constant part $\frac{V_m I_m}{2}\cos\phi$

Double frequency component $\frac{V_m I_m}{2}\left[\cos(2\omega t - \phi)\right]$

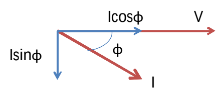
AC through Series RL Circuit

$$\text{Average Power} = \frac{V_m I_m}{2}\cos\phi$$

$\text{Power} = VI\cos\phi$ $\cos\phi = \text{Power Factor}$

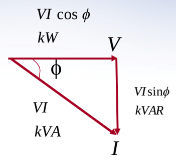


$$\cos\phi = \frac{R}{Z}$$



Power

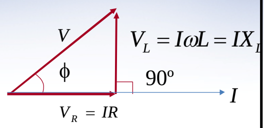
- ❖ Apparent Power
  - ❑ Symbol- S
  - ❑ S=VI
  - ❑ Unit- VA (Volt Ampere) or kVA
- ❖ Active Power
  - ❑ Symbol – P
  - ❑ P=VI cosφ
  - ❑ Unit- W or kW
  - ❑ Power dissipated in resistive circuit
- ❖ Reactive Power
  - ❑ Symbol – Q
  - ❑ Q=VI sinφ
  - ❑ Unit – VAR (Volt Ampere Reactive) or kVAR



Power in AC Circuit

Let Voltage be the reference $\bar{V} = V\angle 0$

$\hat{Z} = Z\angle\phi \Rightarrow$ *inductive* load; then $\bar{I} = I\angle -\phi$

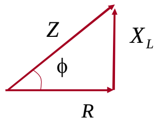


$= I^2 Z\angle\phi = IZ(I\cos\phi + jI\sin\phi)$

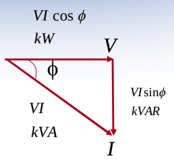
$= VI(\cos\phi + j\sin\phi) = P + jQ$

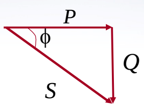
where $P = VI\cos\phi,\quad Q = VI\sin\phi$

$S = P + jQ$



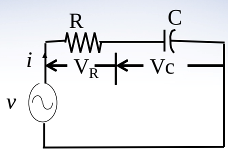
Power Triangle



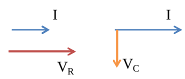


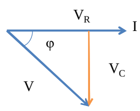
AC through a series RC Circuit

- ❖ v
- ❖ V
- ❖ I
- ❖ V_R= IR
- ❖ V_C= IX_C



$$\frac{\bar{V}}{\bar{I}} = \frac{V\angle 0}{I\angle 90^0} = -jX_C$$





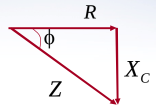
AC through a series RC Circuit

$$\overline{V} = \overline{V}_R + \overline{V}_C$$
$$V = \sqrt{V_R^2 + V_C^2}$$
$$V = \sqrt{(IR)^2 + (IX_C)^2}$$
$$V = I\sqrt{(R)^2 + (X_C)^2}$$
$$V = IZ$$
$$Z = \sqrt{(R)^2 + (X_C)^2}$$

$$\overline{V}_R = \overline{I}R + j0$$
$$\overline{V}_C = 0 - jI\overline{X}_C$$

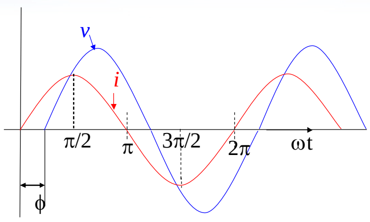
AC through a series RC Circuit

$$\hat{Z} = (R - jX_C) = Z\angle -\phi$$
$$= Z(\cos\phi - j\sin\phi)$$
$$R = Z\cos\phi;\ X_C = Z\sin\phi$$
$$\text{or}\quad Z = \sqrt{(R^2 + X_C{}^2)};$$
$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{1}{\omega CR}\right)$$



AC through a series RC Circuit

$$i = I_m \sin\omega t$$
$$v = V_m \sin(\omega t - \phi)$$



- The current Leads behind the voltage by a phase angle ϕ

Power in series RC and RL Circuit

❖ Derive it

- Write down the frequency, rms and peak values of a voltage wave expressed as v=14.1sin1000πt. Write down the expressions for current flowing when this voltage is applied across a)5Ω resistor, b) 1mH inductor and c)150μF capacitor

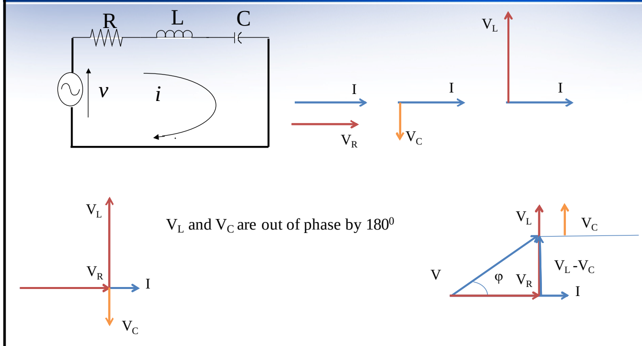
- A coil has an inductance of 20mH and a resistance of 5Ω. It is connected across a supply voltage of v=50sin314t. Obtain similar expression for current

- In a given RL circuit, R=3.5Ω and L=0.1H. Find a) current through the circuit b) pf of a 50Hz voltage V=220<30 is applied across it

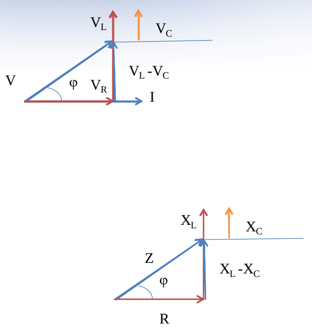
- An ac voltage (80+j60) volts is applied to a circuit and current flowing is (-4+j10) amperes. Find 1) impedance of the circuit and 2) power consumed and phase angle

❖ A current of 5A flows though a non inductive resistance in series with a coil when supplied at 250V, 50Hz. If the voltage across the resistance is 125V and across the coil 200V. Calculate 1) impedance, resistance, and reactance of coil 2) power absorbed by the coil 3) total power. Draw Phasor diagram

AC through Series RLC Circuit



AC through Series RLC Circuit



$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

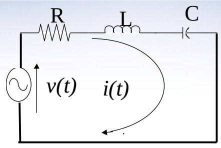
$$V = I\sqrt{(R)^2 + (X_L - X_C)^2}$$

$$V = IZ$$

$$Z = \sqrt{(R)^2 + (X_L - X_C)^2}$$

AC through Series RLC Circuit

- ❖ Let i(t) = I_m sinωt
- ❖ Impedance, Z=R+j(X_L-X_C)
- ❖ (i) If X_L=X_C , resistive circuit
- ❖ (ii) If X_L>X_C, RL series circuit
- ❖ (iii) If X_L<X_C, RC series circuit



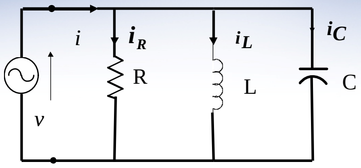
Note: - i(t) is the reference

Circuit Elements	Impedance Z	Phase Angle ϕ
R	R	0°
C	X_C	− 90°
L	X_L	+ 90°
R C	$\sqrt{R^2 + X_C^2}$	Negative, between − 90° and 0°
R L	$\sqrt{R^2 + X_L^2}$	Positive, between 0° and 90°
R L C	$\sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

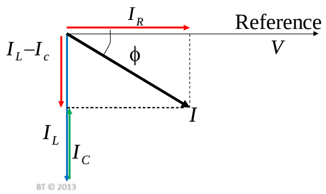
Parallel RLC Cicruit

$v = V_m \sin \omega t$

$\bar{V} = V\angle 0$



- **Phasor Diagram -**



$\bar{I}_R = \frac{V}{R}\angle 0\ ;\ \bar{I}_L = \frac{V}{X_L}\angle -90\ ;$

$\bar{I}_C = \frac{V}{X_C}\angle 90$

Parallel RLC Cicruit

For any parallel circuit,

$$Y = \frac{1}{Z} = \text{Admittance}$$

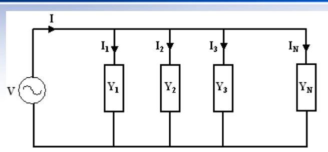
$$= G \pm jB$$

Unit is Siemens

Where G = Conductance

B = Susceptance

Parallel RLC Cicruit



$$Y_1 = \frac{1}{Z_1}; Y_2 = \frac{1}{Z_2}; ......Y_N = \frac{1}{Z_N}$$

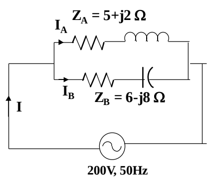
$$Y_{eq} = Y_1 + Y_2 + ... + Y_N = \frac{1}{Z_{eq}} = G_{eq} \pm jB_{eq}$$

$$I_1 = VY_1; I_2 = VY_2; ....I_N = VY_N$$

$$I = I_1 + I_2 + ... + I_N = VY_{eq}$$

Problem

Two circuits Z_A = **5+j2** Ω and Z_B = **6 – j8** Ω are in parallel across a 200V, 50 Hz supply. Find current in each branch, total current, power factor and power consumed.



Ans: $\bar{I}_A = 37.14\angle -21.8°A$ $\bar{I}_B = 20\angle 53.13°A$

$I = 46.54\angle 2.72°A$ cosϕ = **0.9988 lead**

Power = 9.297W